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The formation and maintenance of a leading-edge vortex during the forward motion of an animal wing

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A simple model is presented to explain the observed generation of a quasi-steady vortex at the leading edge of an animal wing that rotates in a horizontal plane about a body-centred axis. Vorticity formed by separation at the leading edge is transported outwards by a spanwise velocity field generated by two sources of spanwise pressure gradient, one induced centrifugally and the other by variations in the vortex size and circulation. The vorticity is then deposited into a trailing vortex system that takes the form of a downward propagating vortex ring. This mechanism appears to apply generally to flying animals but is modelled here for those in hovering flight.

1. Introduction

In Maxworthy (1979, 1981) it was proposed that, in hovering flight, during the forward stroke of the wings at an angle of attack, a quasi-steady leading-edge vortex with a strong axial flow component was formed that substantially modified the airflow over the wing and the forces acting upon it. Since that time a number of other authors have performed experiments that confirm and enlarge upon this observation (e.g. Ellington *et al.* 1996; van den Berg & Ellington 1997; Birch, Dickson & Dickinson 2004). From all these observations, and others, it was clear that as the wing moved forward the flow separated from the sharp leading edge, generating vorticity that was transported spanwise along the leading-edge vortex and eventually deposited into a wing-tip vortex and thus removed from the neighbourhood of the wing, leaving a quasi-steady vortex at the leading edge. This is in contradiction to the dynamical processes that occur in flow over a two-dimensional wing in which the vorticity is removed locally by unsteady vortex shedding, as in the classical von Kármán vortex street. The closest analogous system to the present one is that which occurs over a delta wing at angle of attack. In that case the axial flow in the conical vortex, which is needed to stabilize it, is generated by the component of the approach flow that is in the same direction as the swept leading edge. For the insect wing this not the case and so one needs a different mechanism to generate the axial flow. Previously (e.g. Maxworthy 1981) it has been noted that this mechanism must be related to the centrifugal forces generated by the rotation of the wing about its point of attachment at the insect body, but this has never been quantified, as far as this author is aware. Here an attempt is made to correct this situation and present a simple model that can explain how the axial flow can be set-up and maintained by both the centrifugal force, as outlined above, and the variation in vortex properties with distance from the rotation axis. Both of the vortex types mentioned above are of a class, which also

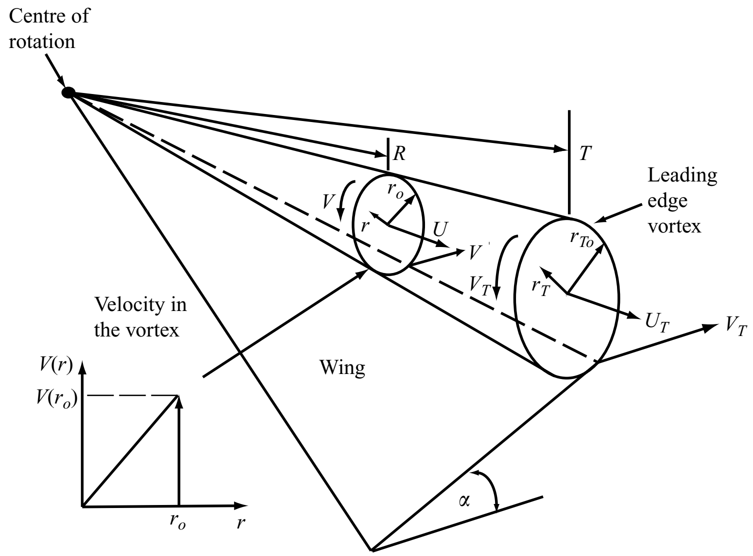

FIGURE 1. Sketch of rotating, triangular wing with attached leading-edge vortex (LEV).

includes tornado-like flows, commonly called Batchelor vortices (Batchelor 1964) in which the axial vorticity and axial velocity occupy the same region in space. Thus once, as here, the location of the axial vorticity is prescribed, the location of the axial velocity must be the same.

2. A dyamical model of the leading-edge vortex on a wing rotating about a fixed vertical axis

In figure 1 the essence of the model is sketched. For simplicity a slender conical flow field is assumed with a wing, of triangular planform, rotating about a centre located at the insect body. The slenderness assumption allows one to make a locally cylindrical approximation to the flow field and greatly simplifies the analysis. The wing has a semi-span, T, and moves at an angle of attack, α, to the direction of wing motion. The wing tip moves with a velocity V_T so that at a radial position, R, the local velocity of the leading edge, V, is simply $V_T R/T$. The vortex grows as vorticity, generated at the leading edge by separation, both accumulates and is transported along the wing, so that finally a quasi-steady state is achieved with a conical vortex, as shown in the figure. At the arbitrary location, R, the vortex has an outer radius, r_o, while the outer radius at the wing tip is r_{To}, so that $r_o = r_{To} R/T$. Within the vortex, along cones with apexes at the centre of rotation, $r/r_o = r_T/r_{To}$. As described later a velocity, U, is induced along the vortex which removes vorticity from where it is generated to the wing-tip and into a trailing vortex ring, which, for simplicity, is not shown on the figure. A continuity argument suggests that the flow velocity around the outer edge of the vortex must be of the same order as, but not equal to, V, the local leading-edge translational velocity, everywhere, in particular being order V_T at the wing-tip. A further assumption is needed, namely that the vorticity within the vortex, ω, is uniform everywhere with a value $2V/r = 2V_T/r_T$, as shown in figure 1. It is a straightforward matter to consider other vorticity distributions but this complicates the simple argument put forth here and adds no substantial, new understanding.

The assumption that the velocity at the edge of the core is of order V leads to the result that the pressure there must be of the order of the upstream atmospheric pressure, p_a. Clearly both assumptions are considerable simplifications of the true state since both must vary around the core in a non-trivial manner, see e.g. Pullin (1978). However, using these assumptions in trying to generate the simplest possible model that has the basic mechanisms correctly identified leads one to estimate the pressure distribution within the core, $p(r)$, to be

$$\begin{aligned} p &= p_a + \rho\omega^2\left(r^2 - r_o^2\right)/8 = p_a + \rho\left(r^2 - r_o^2\right)V^2/2r_o^2 \\ &= p_a + \rho\left(\left[r^2/r_o^2\right] - 1\right)V^2/2 = p_a + \rho\left(\left[r_T^2/r_{To}^2\right] - 1\right)V_T^2R^2/2T^2 \end{aligned}$$

where ρ is the air density.

From a dynamical point of view the important quantity that generates an axial flow is the pressure gradient:

$$\mathrm{d}p/\mathrm{d}R = -\rho V_T^2 R\left(1 - \left[r_T^2/r_{To}^2\right]\right)/T^2.$$

Thus the pressure along the core varies is such a way as to induce an axial velocity, U, in the direction shown in the figure. Before this velocity can be estimated we note that there is another pressure gradient force acting on the vortex. The whole vortex is rotating around the centre shown on the figure. This in turn induces a pressure gradient to balance the centrifugal force acting on all the particles of the vortex that rotate with the wing. This pressure gradient can be estimated to be

$$\mathrm{d}p/\mathrm{d}R = -\rho V^2/R = -\rho V_T^2 R/T^2.$$

It acts over the whole of the vortex cross-section. Thus the total pressure gradient acting along the vortex is

$$\mathrm{d}p/\mathrm{d}R = -\rho V_T^2 R/T^2\left[1 + \left(1 - \left[r_T^2/r_{To}^2\right]\right)\right].$$

Assuming a balance between the pressure gradient and the axial, i.e. spanwise, inertia force, $U\partial U/\partial R = O(U^2/R)$, in the core, the velocity U is then given approximately by solving

$$U^2/R = \left|-V_T^2 R/T^2\left[1 + \left(1 - \left[r_T^2/r_{To}^2\right]\right)\right]\right|$$

so that

$$U = V_T R/T\left[1 + \left(1 - \left[r_T^2/r_{To}^2\right]\right)\right]^{1/2}.$$

The radial distribution function in the outer square brackets is shown in figure 2.

One further result which is of interest concerns the swirl angle of the streamlines since this is intimately related to the criticality of the flow and its ability to generate a vortex breakdown, see e.g. Benjamin (1962). Defining the swirl angle, σ, as that between the local direction of a streamline and the axial direction and using the result that, by assumption, $V = [V_T R/T]r_T/r_{To}$ one obtains

$$\sigma = \arctan\left[r_T/r_{To}\right]/\left[1 + (1 - \left[r_T^2/r_{To}^2\right])\right]^{1/2}.$$

Thus σ varies from zero at $r/r_o = r_T/r_{To} = 0$ to 45^o at $r/r_o = r_T/r_{To} = 1$. Fortuitously, in the paper by Birch *et al.* (2004), photographs of the streamlines are shown in which the central streamline is essentially straight, i.e. follows the curve of the leading edge so that $\sigma = 0^o$, and the outer streamline has swirl angles angles that vary from about 40° to 60°, values which encompass our estimate. These values, as well as the present estimate, are in the range where one would expect a vortex breakdown to be generated. Sarpkaya (1971) gives the critical swirl angle just ahead of a bubble type of

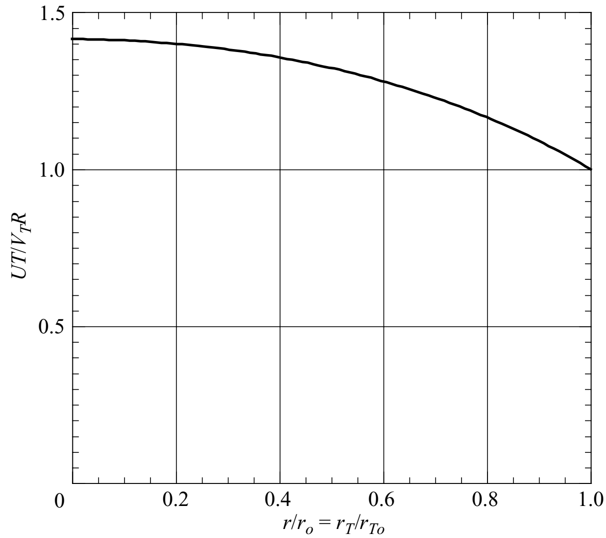

FIGURE 2. Axial velocity function in the vortex core versus radius.

breakdown as 50°, essentially independent of Reynolds number. There is a suggestion of such a structure in Birch *et al.*'s (2004) figure 7, the second and third photographs of sequence D, where the Sarpkaya (1971) critical angle is closely approximated just ahead of the observed vortex breakdown-like structure.

3. Discussion and conclusions

The very simple model presented above shows that the flow generated by a wing rotating about a fixed centre is capable of generating both a rolled-up vortex sheet and a concomitant outward flow along the vortex central axis. The present argument for the latter is partially based on a core that rotates as a solid body around this spanwise axis. While this is certainly the simplest model it is clear that any vorticity distribution that results in a velocity distribution that decreases from a maximum near its outer edge to zero at the centre and increases in size and magnitude, more-or-less conically, will give a similar result. On the other hand the pressure gradient within the particles in any fluid volume that rotates about a centre will of necessity be of a sign to give an outward axial velocity no matter what its state of internal rotation, as in the boundary layer flow over a rotating disk, for example. Thus these two effects give a flow field that generates vorticity at the leading edge and then removes it spanwise to be deposited into a trailing vortex which, as shown in Maxworthy (1979), results in the formation a downward propagating vortex ring that contains the impulse generated by the forces acting on the wing. The present estimate of vortex circulation is the same as that given in Maxworthy (1979) so that the force estimates given there apply to the present case as well.

As discussed in the Introduction the mechanism presented here gives a flow field that is similar to the steady vortex over a delta wing at angle of attack. However, in that case the axial flow is generated by the component of the approach velocity in the direction of the swept leading edge. As demonstrated in van den Berg & Ellington (1997), the flow fields in the two cases are virtually identical.

While the model presented here was formulated for a wing moving in a horizontal plane at an angle of attack, as in figure 1, the same mechanisms are at play when the animal is moving forward and the wings are beating in a plane that is vertical or near-vertical. There the animal probably adjusts its angle of attack with respect to the oncoming flow so that the flow still separates at the leading edge and, as here, there is a variation in vortex properties from wing root to tip as well as general spanwise pressure gradient due to wing rotation.

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